ECONOMICALLY OPTIMAL LEVELS OF CONSERVATION: SENSITIVITY TO UNCERTAIN ECONOMIC CONDITIONS

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ABSTRACT

Optimal levels of conservation are dependent on several variables, some for which values are selected by investors, others that are beyond the control of investors. In this paper, the sensitivity of optimal levels of ceiling insulation to selected economic variables is used to illustrate the importance of these variables. The variables studied include the investor's discount rate, the useful lifetime of the conservation measure, the fuel-price inflation rate, and the timing of unexpected changes in fuel-price inflation. The results show that all the variables have significant effects and that optimal insulation thicknesses can vary by more than a factor of two for reasonable ranges of values for these variables. Unexpected changes in fuel-price inflation, such as the changes that occurred in the early 1980s, can significantly change the optimal level of conservation, particularly when they occur soon after installation.

INTRODUCTION

The economically optimal level of investment in energy-conservation technologies for buildings is sensitive to a number of variables, some that are selected by the investors or are characteristics of the technology used and others that are beyond the control of investors. Investors establish the required rate of return and, within the limits of available capital, the size of acceptable investments. The performance characteristics and durability of energy-conserving devices determine the energy savings and physical lifetimes of the devices. The savings resulting from use of some energy-conservation technologies (e.g., operable shading devices for windows) may depend on user participation. Inadequate care and maintenance may decrease savings and shorten device lifetimes. The integrity of other components of the building also may affect the performance of conservation devices. For example, glass- or mineral-fiber insulation installed in an attic beneath a leaky roof will, at best, perform inadequately, and, at worst, may be damaged extensively enough to require removal and replacement.

Factors beyond the control of users and not inherent in the technology can have comparable or even greater effects on the economics of energy conservation. In recent years, world oil prices have decreased, and the escalation of most residential energy prices has slowed (see Figure 1). Between 1977 and 1981, average prices for residential heating fuels in the U.S. increased at average rates of 27.3%/yr for fuel oil and 16.1%/yr for natural gas. Since 1981, the average fuel-oil price has decreased by 13.5%. Residential natural-gas prices continued to increase at greater than 15%/yr during 1982 and 1983, but since then they have increased at less than 2%/yr and may now be decreasing, even as decontrol continues to be phased in. These changes in fuel prices were unforeseen in the late 1970s. Because energy prices determine the value of energy saved, such unexpected changes can have significant effects on the economics of conservation. Other variables beyond the control of users also have important effects. Tax

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rates and structures and the availability of tax credits for conservation influence to varying degrees the economics for individual investors. And general economic conditions affect expectations concerning interest rates and inflation and, therefore, affect investors' discount rates.

In this paper the sensitivity of the economically optimal level of ceiling insulation in residences and other small buildings is used to illustrate the effects of several key economic variables, including the discount rate, the fuel-price inflation rate, and device lifetimes. Emphasis is placed on the importance of fuel-price inflation. Several price-escalation scenarios are considered, some with time-dependent price inflation rates, in order to illustrate the effects of unexpected price changes, like the leveling of fuel prices in the past few years. For simplicity, the building considered is not air conditioned. Although the numerical results do not apply to air-conditioned buildings, the qualitative effects of changes in the economic variables are unchanged.

ENGINEERING-ECONOMIC MODEL

Determination of the optimal level of ceiling insulation requires estimates of energy savings as a function of insulation level, the cost of insulating as a function of the amount of insulation, and the value of saved energy as a function of time over the useful life of the insulation. Because cash flows occur at different times, the net present value of the savings obtained over the useful lifetime is most appropriate for evaluating the economic benefits of the insulation.

Energy Savings

The rate of heat loss per unit area through the ceiling (Q/A) to an unheated attic may be estimated from the relation

$$Q/A = (T_i - T_a)/R, \tag{1}$$

where

Q = rate of heat loss through the ceiling

A = total ceiling area

T_i = inside air temperature

T_a = attic air temperature

R = thermal resistance of the ceiling

The attic air temperature is given by the relation (ASHRAE, 1985):

$$T_a = [A_cU_cT_i + T_o(KA_cV_a + A_rU_r + A_wU_w + A_gU_g)] \div [A_c(U_c + KV_a) + A_rU_r + A_wU_w + A_gU_g] \tag{2}$$

where

A_c, A_r, A_w = areas of the ceiling, roof, and side-wall of the attic, respectively

U_c, U_r, U_w = overall coefficients of heat transfer for the ceiling, roof, and side-wall of the attic

V_a = ventilation rate for the attic

K = 1.08 for V_a in cfm/ft^2 and 1200 for V_a in L/s·m^2

The value of U_c is a function of the amount of ceiling insulation installed, and, as a result, the attic temperature also depends on the insulation level. However, for $R > 11$ [i.e., $U_c < 0.091$ Btu/hr·ft^2·F(0.516 W/m^2·°C)], T_a is only a weak function of R. Therefore, in finding the optimal value of R, the dependence of T_a on R may be neglected without introducing substantial errors. For example, for an average size house with a 1500-ft^2 floor area, an increase in R from 11 to 30 increases the driving temperature difference, $T_i - T_a$, by less than 7.5%. Therefore, the heat loss through the ceiling is, to a good approximation, proportional to 1/R.

The annual heat loss (Q_o/A) through the attic before treatment (i.e., the addition of insulation) may be estimated by using Equation 1, with the temperature difference replaced by the number of degree hours per year associated with the difference between the indoor and attic temperatures, which may be obtained from a temperature-bin analysis. The heat loss (Q/A) through the attic after insulation is added is then given by

$$Q/A = (Q_o/A)R_o/R. \tag{3}$$

The factor R_o/R is the ratio of the R-value before treatment (R_o) to the R-value after installation of additional blow insulation (R).

The corresponding decrease in heat loss ($\Delta Q/A$) is

$$\Delta Q/A = (Q_o/A) - (Q/A) = (Q_o/A)\ (1 - R_o/R). \tag{4}$$

Because this paper addresses the qualitative effects of changes in economic variables on optimal levels of conservation, we consider only one conservation option, namely, the installation of blown glass-fiber attic insulation. The R-value per unit thickness for blown glass-fiber insulation is about 2.2 hr·ft^2·F/Btu·in (in inch-pound units; BNL 1978). The precise value varies and can depend on density, fiber thickness, and compaction during installation. Because all optimal thicknesses calculated in the study exceeded the width of 2 x 4 ceiling joists, glass-fiber insulating batts were assumed already in place, giving an average R-value for the ceiling [including the joists spaced at 16-in (0.41-m) intervals on center] of R_o = 11.41. Therefore, the R-value was estimated from the relation

$$R = R_o + bx \tag{5}$$

<u>where</u>

R_o = R-value before treatment = 11.41 for the selected initial case

b = R-value per unit thickness of added insulation

= 2.2 for blown glass-fiber insulation

x = thickness of additional insulation

Upon substitution, Equation 4 for the decrease in heat loss becomes

$$\Delta Q/A = (Q_o/A)[1 - R_o/(R_o + bx)] \tag{6}$$

Value of Savings

The estimated monetary savings (S_n/A) in any year, n, resulting from installation of ceiling insulation are given by the equation

$$S_n/A = (\Delta Q/A)\ P_n/\eta, \tag{7}$$

<u>where</u>

S_n/A = annual monetary savings per unit area in year n

P_n = price of heating fuel in year n

η = seasonal average heater efficiency

The fuel price may be projected by using the relations

$$P_n = P_o\ (1 + i_{f1})(1 + i_{f2})\cdots(1 + i_{fn}), \tag{8}$$

in general, and

$$P_n = P_o\ (1 + i_f)^n \tag{9}$$

for constant fuel-price inflation,

<u>where</u>

P_o = fuel price at the time of installation (usually the present)

i_{fj} = fuel-price inflation rate in year j, for j = 1,2,3,..., n

i_f = constant effective fuel-price inflation rate between the present and year n

The present value of the heating-fuel savings over the useful life of the installation is given by

$$PV(\text{savings}) = (\Delta Q/A)\ P_o \sum_{j=1}^{L} [(1 + i_{fj})/(1 + d)]^j/\eta, \tag{10}$$

<u>where</u>

d = investor's discount rate.

For a constant fuel-price inflation rate (i.e., $i_f = i_{f1} = i_{f2} = \cdots = i_{fn}$),

$$PV(\text{savings}) = (\Delta Q/A)P_o[(1 + i_f)/(d - i_f)] \times \{1 - [(1 + d)/(1 + i_f)]^{-L}\}/\eta \qquad \text{for } i_f \neq d \tag{11}$$

and

$$PV(\text{savings}) = (\Delta Q/A)P_oL/\eta \qquad \text{for } i_f = d. \tag{12}$$

For simplicity, we assume that the cost of the insulation is paid in full at the time of installation. As the result,

$$PV(\text{insulation}) = C_i, \tag{13}$$

<u>where</u>

PV(insulation) = present value of the installed insulation

C_i = total cost at the time of installation

Use of a loan to finance the installation can readily be included by determining the loan payments and discounting each payment to the present. The net effect of financing the purchase with a loan is to decrease the present value of the insulation costs, because money will only be borrowed if the interest rate on the loan is less than or equal to the investor's discount rate. Because the primary purpose of this paper is to show the qualitative effects of unexpected changes in fuel price, loans are not considered here.

Insulation costs can be expressed as the sum of fixed costs that are independent of the amount of insulation installed and variable costs that are roughly proportional to the amount installed. The cost (across several contractors) of blown glass-fiber ceiling insulation installed in the attic of an existing house in St. Louis during the late summer of 1985 can be approximated by the linear relation

$$C_i = C_f + C_v\ x \tag{14}$$

where

C_f = fixed cost per unit area = $\$0.004/ft^2$ ($\$0.043/m^2$)

C_v = variable cost per unit area per unit thickness = $\$0.033/ft^2 \cdot in$ ($\$13.98/m^3$)

x = insulation thickness

The net present value (NPV) of the insulation is the difference between the present value of the savings and the present value of the cost of installing the insulation, i.e.,

$$NPV = PV(savings) - PV(insulation). \tag{15}$$

Using Equation 11 or 12 for the present value of savings with Equation 6 used for $\Delta Q/A$ as a function of insulation thickness, and Equation 13 for the present value of the costs with Equation 14 used to represent the cost as a function of insulation thickness, Equation 15 for the NPV can be rewritten as

$$NPV = (Q_o/A)[1 - R_o/(R_o + bx)]\ P_o\ [(1 + i_f)/(d - i_f)] \times \{1 - [(1 + d)/(1 + i_f)]^{-L}\}/\eta - (C_f + C_v x) \quad \text{for } i_f \neq d, \tag{16}$$

and

$$NPV = (Q_o/A)[1 - R_o/(R_o + bx)]L/\eta - (C_f + C_v x) \quad \text{for } i_f = d. \tag{17}$$

Other forms that incorporate variable fuel-price inflation rates can be derived by using Equation 10 for the present value of the savings.

Optimal Insulation Level

Insulating at the optimal level results in the maximum net present value of the savings. The optimum also corresponds to the thickness for which the marginal savings equal the magnitude of the marginal costs. Other life-cycle cost criteria (e.g., maximum annual equivalent worth) for identifying the optimal insulation thickness provide the same result. The maximum NPV occurs for the insulation thickness, x, for which $d(NPV)/dx = 0$ and $d^2(NPV)/dx^2 < 0$. Therefore, from Equations 16 and 17, the optimal thickness (x_{opt}) is given by the relation

$$x_{opt} = [R_o(Q_o/A)\ P_o\ D/(\eta\ b\ C_v)]^{1/2} - R_o/b, \tag{18}$$

where

D = series present worth factor with price escalation

$= [(1 + i_f)/(d - i_f)]\ \{1 - [(1 + d)/(1 + i_f)]^{-L}\}$ for $i_f \neq d$

$= L$ for $i_f = d$

Equation 18 may be used to determine the optimal insulation thickness for variable fuel-price escalation rates by using appropriate relations for D developed from the series in Equation 10.

REPRESENTATIVE RESULTS

Optimal thicknesses of blown glass-fiber ceiling (i.e., attic) insulation installed over R-11 batts have been determined for several combinations of discount rate, fuel-price inflation scenario, and useful lifetime. Values of other variables that were maintained constant throughout the analysis are shown in Table 1. Weather conditions correspond to average weather for St. Louis, MO, which is about average for winters in the U.S. (about 5,000 degree-days per year, similar to New York City, Columbus, OH, and Philadelphia, PA; see ASHRAE 1981, p. 24.23). Before installation of blown insulation, the ceiling consists of 1/2-in (12.7-mm) gypsum board on 2 in x 4 in (50.8 mm x 101.6 mm) wood ceiling joists. The space between the joists is filled with R-11 fiberglass batts. All optimal insulation levels for the specified weather conditions exceed R-11. A heater efficiency of 70% is used in all calculations. During the late 1970s, the average seasonal efficiencies for gas- and oil-fired residential heating systems were about 50-60% (OTA 1979). Newer furnaces generally have higher efficiencies, and recently, several high-efficiency furnaces have entered the market with efficiencies exceeding 90%. Because new designs require many years to significantly penetrate the market, an estimate of 70% is used in this analysis. Actual efficiencies of existing residential heating systems may range from well below 40% for older systems to greater than 90% for new high efficiency systems. Optimal insulation levels are strongly dependent on heating-system efficiencies because fuel consumption (and, fuel expenditures) are inversely proportional to seasonal average efficiency. Because this study addresses only the impact of economic variables, the efficiency is held constant. The costs for additional blown glass-fiber insulation correspond to estimates given by several contractors in St. Louis during late summer of 1985. Although cost estimates vary somewhat, the linear function given by Equation 14 is a good approximation to reported costs for retrofits. The initial fuel price of $6/10^6 Btu ($5.69/GJ) is slightly greater than the average residential natural-gas price in the U.S. in 1984 and the first quarter of 1985, and about 80% of the current average residential fuel-oil price.

Optimal thicknesses for the additional blown insulation and corresponding R-values of the added insulation (not including the R-11 batts originally in place) are shown in Figures 2 and 3 as functions of the fuel-price inflation rate for four selected discount rates and for useful lifetimes of 7 and 20 years, respectively. The optimal thickness is relatively sensitive to both i_f and d. For example, for a seven-year lifetime and a discount rate of 15%, x_{opt} is about 2.7 in (68.6 mm) for i_f = 0.5 and about 4.2 in (107 mm) for i_f = 0.15, the difference being 56%. As a result, if the average fuel-inflation rate were estimated at 15% while the rate was actually 5%/yr, the monetary investment in ceiling insulation would exceed the optimal value by about 53% (the difference between 53% and 56% being attributable to the fixed cost representing a few percent of the total cost).

The effect of the discount rate also is shown in Figures 2 and 3. As the discount rate increases, the present value of savings that occur at any time in the future decreases, thus lowering the optimal insulation thickness (compare, for example, curves for d = 0.15 and 0.10). Irrespective of the discount rate, as the ratio i_f/d increases, the slope of x_{opt} versus i_f also increases. As the result, the optimal insulation thickness is most sensitive to estimates of the fuel-price inflation rate for small discount rates. The selection of appropriate discount rates is discussed briefly in the next section.

The estimated useful lifetime (L) also substantially affects the optimal insulation thickness (compare Figures 2 and 3). For i_f = 10%/yr and d = 15%/yr, the optimal insulation thickness is 3.4 in (86 mm) for L = 7 years and 7.6 in (193 mm) for L = 20 years. As i_f increases, the slope of x_{opt} versus i_f increases, and the effect of lifetime becomes even more pronounced.

Abrupt changes in the fuel-price inflation rate, like those that occurred for natural gas and fuel oil during the past few years, also can significantly affect the optimal insulation level. Four representative fuel-price inflation scenarios are shown in Figure 4: A - inflation at a constant rate, i_{f1}; B - inflation at i_{f1} followed by a change to the lower rate, i_{f2}, at time n_1; C - inflation at i_{f1} followed by a constant price after n_1 years (i.e., i_{f2} = 0); and D - inflation at the rate i_{f1}, followed by a constant price for years n_1 to n_2, followed by inflation at the rate i_{f2} after year n_2. The corresponding optimal insulation levels are shown in Figures 5 and 6 as functions of the year n_1 at which the first change in price-inflation rate occurs, for lifetimes of 7 years and 20 years, respectively, and a discount rate d = 10%/yr. For a constant price-inflation rate of 20%/yr over the entire life of seven years, the optimal insulation level is 6.1 in (155 mm); all curves terminate at this thickness because n_1 = L. For an abrupt change to a constant price after year n_1, the decrease in optimal insulation thickness is greatest for changes that occur soon after installation. For L = 7 years, leveling of the fuel price after two years (i.e., n_1 = 2) decreases the optimal thickness by about 33% from 6.1 in (155 mm) to 4.1 in (104 mm). The decreases are even larger for a useful life of 20 years. Optimal insulation levels for scenario D are shown only for L = 20 years (in Figure 6) with the interval of time, $n_2 - n_1$, during which the price is constant, fixed at five years. In all cases the time of the first change in fuel-price inflation rate is more significant than the differences between the scenarios considered.

DISCUSSION

The results show that the value of the discount rate can have a substantial effect on the economically optimal insulation thickness (see Figures 2 and 3), with smaller discount rates leading to larger values of the optimal thickness. Therefore, careful selection of the discount rate is crucial in generic studies in which a single value of the discount rate is used, especially when the results are used to develop prescriptive standards. Often a value of about 5% or 6% is selected for homeowners, using the rationale that the best alternative monetary investment is a savings account with that interest rate. While the discount rate might be equivalent to the best alternative rate of return for a business, this is not necessarily true for individual consumers. By definition, the discount is the rate of interest at which an investor feels adequately compensated for trading money today for money at some time in the future for investments of similar magnitude and risk (Marshall and Ruegg 1980). Because many homeowners are willing to make purchases of a size similar to the investment in additional attic insulation (~$300 to $1000) using credit cards with interest rates of 18% to 22% per annum, a more appropriate value of the discount rate may be 15% to 20%. Of course, the value varies among consumers and depends on individual financial circumstances and personal preferences. Consequently, the optimal level of investment in attic insulation (and other conservation measures) may vary by a factor of two or more among homeowners.

The projected fuel-price inflation rate (i_f) also significantly affects the optimal insulation level. Unanticipated abrupt decreases in i_f substantially lower the economically optimal insulation level. Changes that occur soon after installation of conservation measures have the greatest impact, both because the remaining lifetime over which savings can accrue is greater and because savings nearer the present are discounted less (i.e., have greater present worth per current dollar saved). Homeowners with oil-fired heating systems who in 1980 or 1981 installed extremely high levels of insulation, which appeared optimal if fuel-oil prices continued to escalate at greater than 20%/yr, may have invested much more than the optimum once fuel-oil prices decreased between 1982 and 1985. Hindsight is perfect, and no one can accurately predict the future. However, the "risk averse" consumer may install optimal insulation levels corresponding to a conservatively low estimate of expected fuel-price inflation. If prices continue to escalate at higher rates over the long term, additional insulation may easily be added. The additional cost of installing the insulation in two steps is negligible if the fixed cost accounts for only a few percent of the total cost. The "risk prone" consumer who installs the amount of insulation that is optimal for a very high fuel-price inflation rate cannot easily correct for the overinvestment. The insulation cannot be removed easily, and even if it could, no refund would be available.

Selection of an appropriate estimate of the useful lifetime also has a significant effect on the optimal level of insulation (compare Figures 2 and 3). The physical lifetime of glass fiber insulation may be greater than 20 years; however, the lifetime should be selected to correspond to the time interval over which the investor expects to obtain benefits (i.e., savings) from the conservation measure. For homeowners, this may be closer to the average time between changes of residence in the U.S. of about seven years, rather than the physical lifetime of the insulation of 20 years or more.

No salvage value has been attributed to the insulation in this study. A non-zero salvage value will increase the optimal insulation level somewhat; however, because the benefit associated with the salvage value occurs at termination of the useful lifetime, the present value is much smaller than the salvage value itself. Inclusion of the salvage value is most important for small discount rates and short useful lifetimes. The salvage value is the incremental market value of the home attributable to the insulation at the time of sale. Because this value is small compared to the market value of the house, measurement of salvage value for individual conservation measures is difficult. Although the value associated with several energy-conserving measures may be detectable, to our knowledge, little, if any, reliable data presently exist.

Although not shown in this paper, tax credits significantly increase the optimal insulation level by effectively decreasing the initial capital cost. Tax credits have been neglected here because the purpose of this study is not to develop absolute recommendations for attic-insulation levels but to illustrate sensitivity to selected economic variables. Also, federal income-tax credits for energy conservation are set to expire on January 1, 1986.

CONCLUSION

The major conclusions from this study are:

1. Discount rate, fuel-price inflation rate, and expected useful lifetime, all significantly affect the optimal level of conservation (by as much as a factor of two or greater).

2. Unexpected changes in the rate of fuel-price inflation can cause estimated optimal levels of conservation to lead to significant overinvestment or underinvestment.

3. The "risk averse" consumer will select conservatively low fuel-inflation rates for analyzing investments in conservation to avoid overinvestment, which is not easily corrected. Underinvestment can easily be corrected by installing more insulation (or other devices) in the future when the investor is satisfied that long-term trends have been established.

4. Discount rates vary among consumers because of differences in financial circumstances and personal preferences. As the result, economically optimal levels of conservation may vary by more than a factor of two (even with all other values of estimated variables the same).

5. Although the addition of blown glass-fiber insulation has been used to illustrate the effects of selected economic variables, the qualitative results apply to energy-conservation investment in general.

The following qualifications are necessary to prevent misuse of the results:

1. The absolute values given for optimal levels of insulation correspond only to the specific conditions listed in Table 1, i.e., average weather conditions for St. Louis, MO, costs for retrofit in St. Louis during the summer of 1985, etc.

2. The values given are optima based on heating-fuel savings only. Energy savings associated with air conditioning increase the optimal insulation thickness.

3. Throughout the analysis, sufficient capital is assumed available. When funds are limited, alternative conservation investment opportunities must be prioritized, even if all are cost effective. This may lead to investments that correspond to less than optimal levels based on unlimited funding. However, if the alternatives are prioritized according to net present value, the resulting level of conservation will still be optimal within the constraints of available funds.

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TABLE 1

Values of Fixed Variables

Variable	Symbol	Value
Degree hours per year*	---	106,517 F·h/yr (59,176 °C·h/yr)
Original thermal resistance of the ceiling	R_o	11.41 ft²·h·F/Btu (conventional units)
Original annual heat loss through the ceiling	Q_o/A	9335 Btu/ft²·yr (60.5 x 10^6 J/m²·yr)
Thermal resistance per unit thickness of blow glass fiber insulation	b	2.2 ft²·h·F/Btu-in (conventional units)
Heater efficiency	η	70%
Initial fuel price	P_o	\$6/10^6 Btu (\$5.69/GJ)
Fixed cost of insulation	C_f	\$0.004/ft² (\$0.043/m²)
Variable cost of insulation	C_v	\$0.033/ft²·in (\$13.98/m³)

*Based on the difference between the base temperature of 65 F and the attic temperature. Determined using temperature bins for St. Louis with an attic temperature calculated for each outside temperature bin.

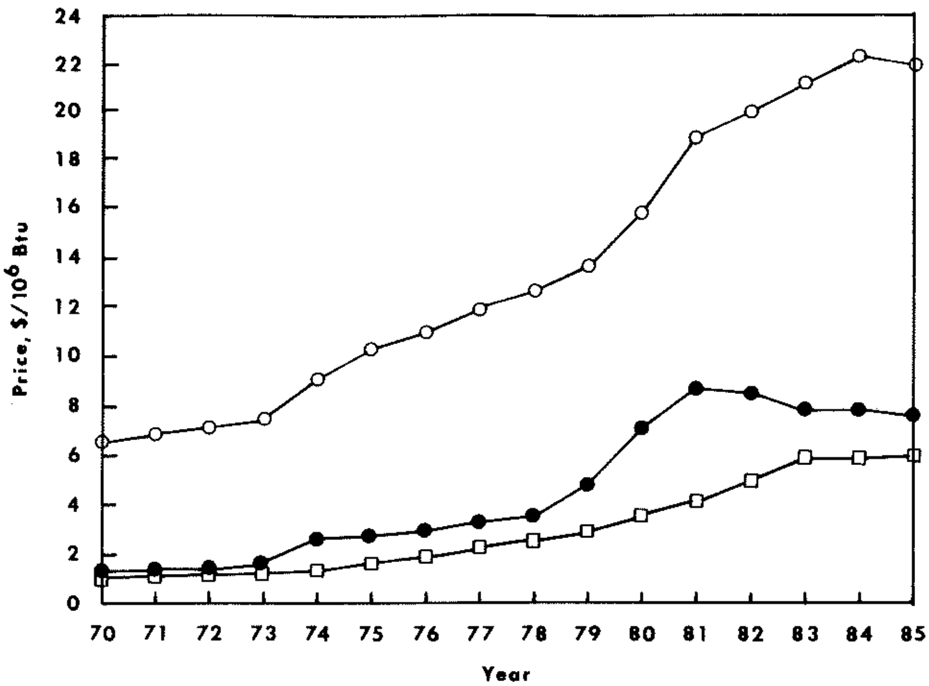

Figure 1. Average residential energy prices in U.S. between 1970 and 1985 for natural gas (□), fuel oil (•), and electricity (○). Slopes of lines connecting data points are equal to rate of fuel-price inflation. Data are from DOE (1985a and 1985b)

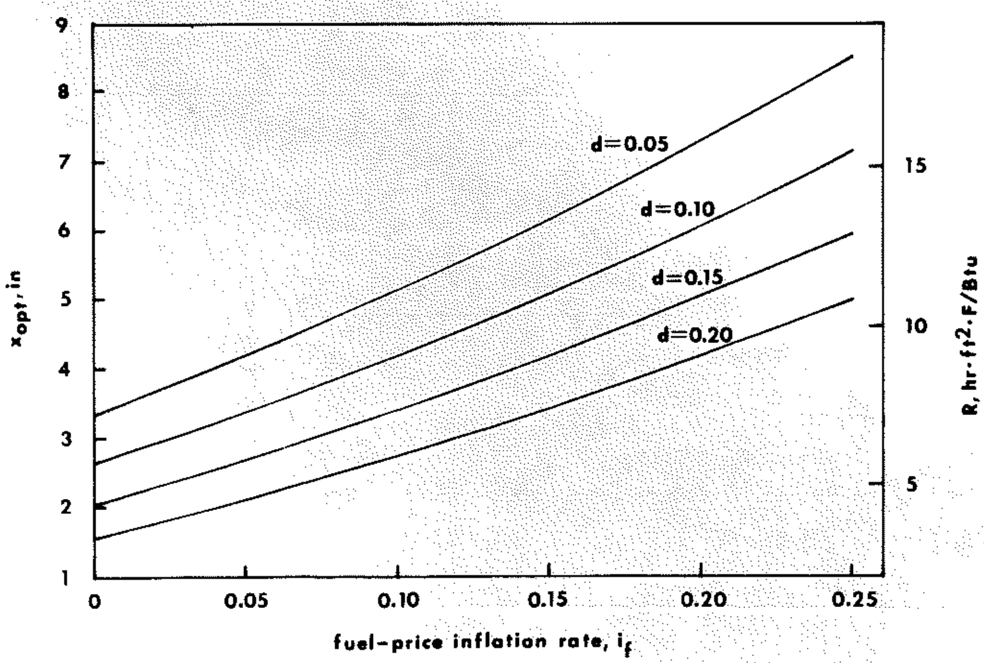

Figure 2. Optimal thicknesses (x_{opt}) of additional blown glass-fiber ceiling insulation and the corresponding R-values as functions of the fuel-price inflation rate (i_f) for four selected discount rates (d). Results correspond to a useful lifetime of seven years

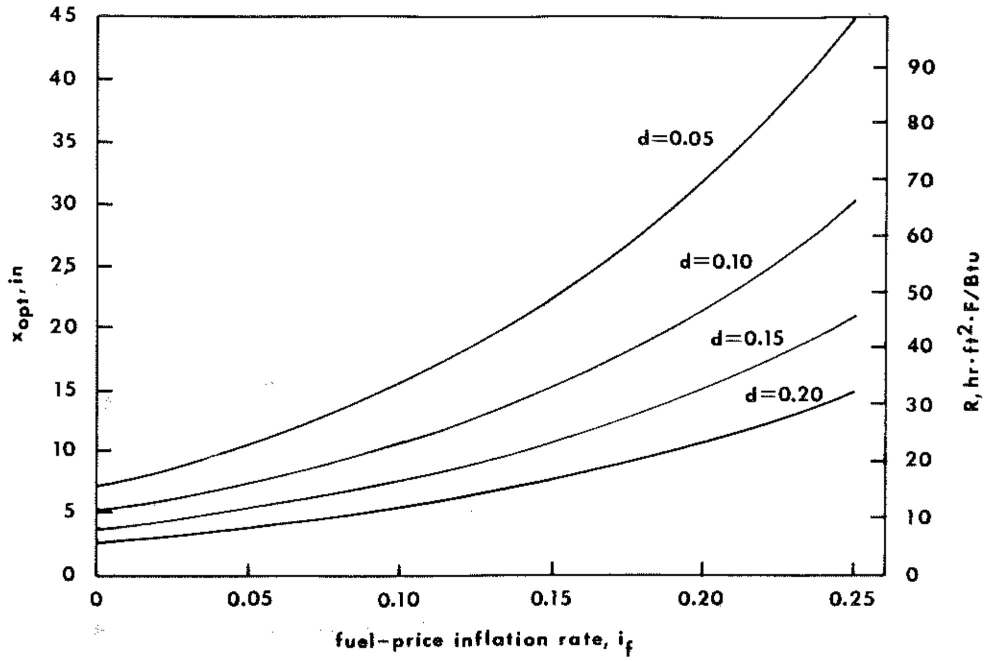

Figure 3. *Optimal thicknesses (x_{opt}) of additional glass-fiber insulation and the corresponding R-values as functions of the fuel-price inflation rate (i_f) for four selected discount rates (d). Results correspond to a useful lifetime of 20 years*

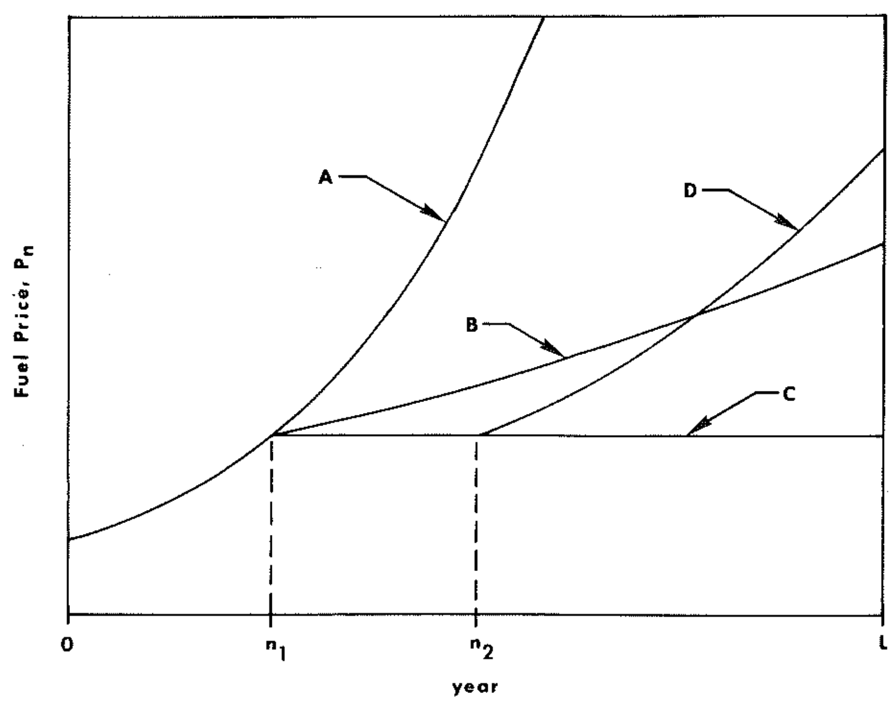

Figure 4. *Scenario for fuel-price inflation: A = inflation at constant rate i_f = 20%/yr; B = inflation at rate i_{f1} = 20%/yr for n_1 years, followed by inflation at 5%/yr; C = inflation at rate i_{f1} = 20%/yr for n_1 years, followed by a constant fuel price; D = inflation at i_{f1} = 20%/yr for n_1 years, a constant rate for five years between years n_1 and n_2, followed by inflation i_{f2} = 10%/yr*

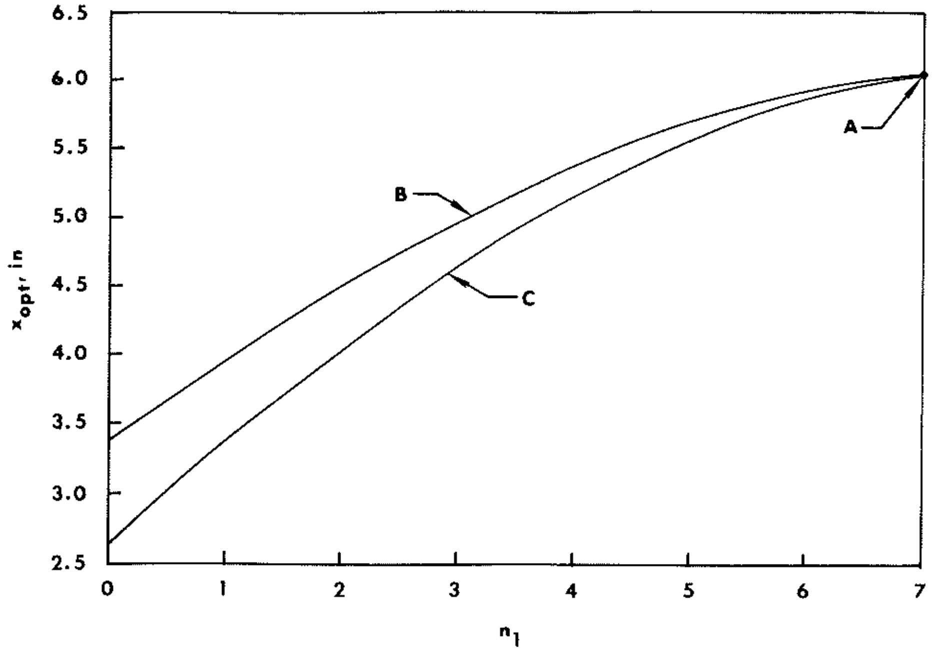

Figure 5. Optimal thicknesses (x_{opt}) of additional blown glass-fiber insulation that correspond to the fuel-price inflation scenarios in Figure 4 are shown as functions of the year n_1 in which the first change in price inflation occurs. Results are for a discount rate of 10% and a lifetime of seven years

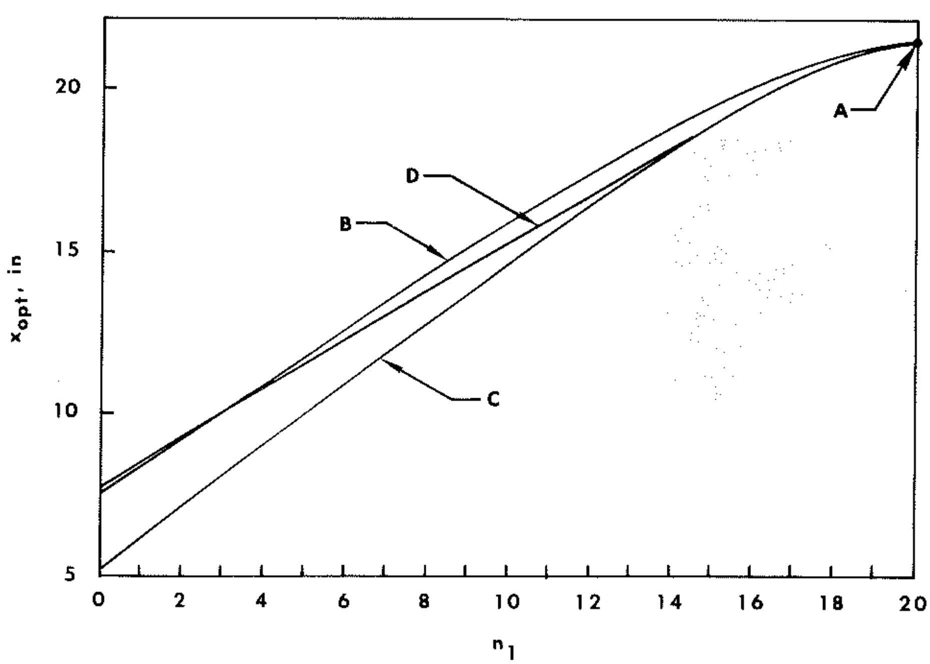

Figure 6. Optimal thicknesses (x_{opt}) of addistional blown glass-fiber insulation that correspond to the fuel-price inflation scenarios in Figure 4 are shown as functions of the year n_1 in which the first change in price inflation occurs. Results are for a discount rate of 10% and a lifetime of 20 years